



théorèmes de l'algèbre de Boole

invariance

$$A \cdot 0 = 0$$

$$A + 1 = 1$$

Élément neutre

$$A \cdot 1 = A$$

$$A + 0 = A$$

idempotence

$$A \cdot A = A$$

$$A + A = A$$

complémentarité

$$A \cdot \bar{A} = 0$$

$$A + \bar{A} = 1$$

involution

$$\bar{\bar{A}} = A$$



Postulats de l'algèbre de Boole

$$0.0 = 0$$

$$0 + 0 = 0$$

$$0.1 = 0$$

$$0 + 1 = 1$$

$$1.1 = 1$$

$$1 + 1 = 1$$

$$\bar{0} = 1$$

$$\bar{1} = 0$$



Lois de l'algèbre de Boole

Commutativité

$$A.B = B.A \quad A+B = B+A$$

Associativité

$$(A.B).C = A.(B.C)$$

$$(A+B)+C = A+(B+C)$$

Distributivité

$$A.(B+C) = A.B + A.C$$

$$A+(B.C) = (A+B).(A+C)$$



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Absorption :

$$A + (A.B) = A$$

$$A.(A + B) = A$$

$$A + A.B = A.1 + A.B = A.(1 + B) = A.1 = A$$

$$A.(A + B) = (A + A).(A + B) = A + (A.B) = A$$

$$A.(A + B) = (A.A) + (A.B) = A + (A.B) =$$





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Expansion

$$A.B + A.\bar{B} = A$$

$$(A + B).(A + \bar{B}) = A$$

$$A.B + A.\bar{B} = A.(B + \bar{B}) = A.1 = A$$

$$(A + B).(A + \bar{B}) = A + (B.\bar{B}) = A + 0 = A$$



Lois de l'algèbre de Boole

$$A + \bar{A} \cdot B = A + B$$

$$A + (\bar{A} \cdot B) = (A + \bar{A}) \cdot (A + B) = 1 \cdot (A + B) = A + B$$



Lois de l'algèbre de Boole

$$A.B + \bar{A}.C + \mathbf{B.C} = A.B + \bar{A}.C$$

$$A.B + \bar{A}.C + B.C$$

$$= AB + \bar{A}C + BC \cdot (\mathbf{A + \bar{A}})$$

$$= AB + \bar{A}C + BCA + BC\bar{A}$$

$$= \mathbf{AB} + \mathbf{ABC} + \bar{A}C + \bar{A}CB$$

$$= \mathbf{AB}(1 + C) + \bar{A}C(1 + B)$$

$$= \mathbf{AB + \bar{A}C}$$



Loi de DE MORGAN

$$\overline{A \cdot B} = \bar{A} + \bar{B}$$

$$\overline{A + B} = \bar{A} \cdot \bar{B}$$