

I. Fonctions Circulaires

Fonction cos et sin

impair: Sin, pair: cos

- $\forall x \in \mathbb{R}, -1 \leq \cos x \leq 1$ et $-1 \leq \sin x \leq 1$
- cos et sin sont périodiques de $T = 2\pi$



- $\cos^2 x + \sin^2 x = 1$
- $(\cos x)' = -\sin x$
- $(\sin x)' = \cos x$
- $\cos(-x) = \cos x$
- $\cos(\pi - x) = \cos(\pi + x) = -\cos x$
- $\sin(-x) = \sin(\pi + x) = -\sin x$
- $\sin(\pi - x) = \sin x$
- $\cos(\frac{\pi}{2} - x) = \sin x$; $\sin(\frac{\pi}{2} - x) = \cos x$
- $\cos(\frac{\pi}{2} + x) = -\sin x$; $\sin(\frac{\pi}{2} + x) = \cos x$
- sin et cos sont $\mathcal{C}^\infty$
- $(\cos x)^{(n)} = \cos(x + n\frac{\pi}{2})$; $(\sin x)^{(n)} = \sin(x + n\frac{\pi}{2})$

Fonction tan et cotan

- $\forall x \in \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi, k \in \mathbb{Z}\}$, $\tan x = \frac{\sin x}{\cos x}$
- $\forall x \in \mathbb{R} \setminus \{k\pi, k \in \mathbb{Z}\}$, $\cotan x = \frac{\cos x}{\sin x}$
- cotan et tan sont périodiques de $T = \pi$



- tan x et cotan x sont $\mathcal{C}^\infty$
- $(\tan x)' = \frac{1}{\cos^2 x} = 1 + \tan^2 x$
- $(\cotan x)' = -\frac{1}{\sin^2 x} = -(1 + \cotan^2 x)$

- $\tan(-x) = \tan(\pi - x) = -\tan x$
- $\cotan(-x) = \cotan(\pi - x) = -\cotan x$
- $\tan(\frac{\pi}{2} - x) = \cotan x$; $\cotan(\frac{\pi}{2} - x) = \tan x$
- $\tan(\frac{\pi}{2} + x) = -\cotan x$; $\cotan(\frac{\pi}{2} + x) = -\tan x$

Formule d'addition

- $\cos(a+b) = \cos a \cos b - \sin a \sin b$
- $\cos(a-b) = \cos a \cos b + \sin a \sin b$
- $\sin(a+b) = \sin a \cos b + \sin b \cos a$
- $\sin(a-b) = \sin a \cos b - \sin b \cos a$
- $\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$
- $\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$

Somme en produit

- $\cos a + \cos b = 2 \cos(\frac{a+b}{2}) \cos(\frac{a-b}{2})$
- $\cos a - \cos b = -2 \sin(\frac{a+b}{2}) \sin(\frac{a-b}{2})$
- $\sin a + \sin b = 2 \sin(\frac{a+b}{2}) \cos(\frac{a-b}{2})$
- $\sin a - \sin b = 2 \sin(\frac{a-b}{2}) \cos(\frac{a+b}{2})$
- $\tan a + \tan b = \frac{\sin(a+b)}{\cos a \cos b}$
- $\tan a - \tan b = \frac{\sin(a-b)}{\cos a \cos b}$

$$\sin(\arccos x) = \sqrt{1-x^2}$$

Formule de duplication

- $\cos 2a = \cos^2 a - \sin^2 a = 2 \cos^2 a - 1 = 1 - 2 \sin^2 a = \frac{1 - \tan^2 a}{1 + \tan^2 a}$
- $\sin 2a = 2 \sin a \cos a = \frac{2 \tan a}{1 + \tan^2 a}$
- $\tan 2a = \frac{2 \tan a}{1 - \tan^2 a}$

Produit en somme

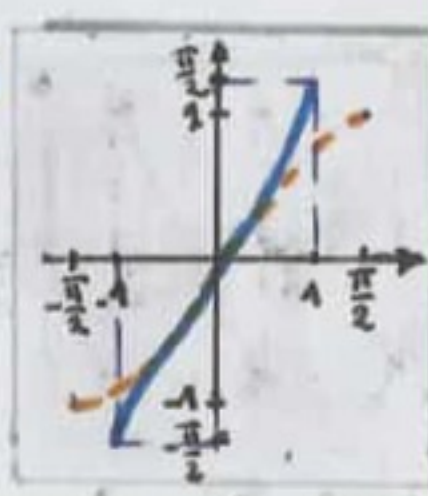
- $\sin a \cos b = \frac{1}{2} [\sin(a+b) + \sin(a-b)]$
- $\cos a \cos b = \frac{1}{2} [\cos(a+b) + \cos(a-b)]$
- $\sin a \sin b = \frac{1}{2} [\cos(a-b) - \cos(a+b)]$

II. Fonctions Circulaires Réciproques

Arcsin

sin: orange, arcsin: blue

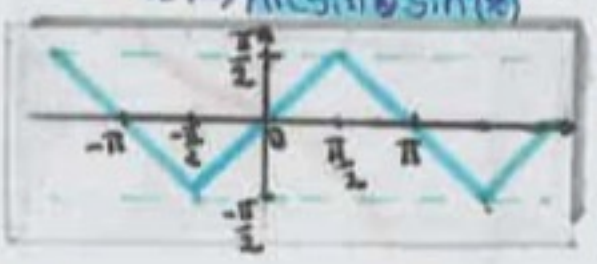
- $\forall x \in [-1, 1], \text{Arcsin}(x) \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
- Arcsin est impaire
- $\sin \circ \text{Arcsin}(x) = x, \forall x \in [-1, 1]$
- $\text{Arcsin} \circ \sin(x) = x, \forall x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
- Arcsin est dérivable sur $] -1, 1 [$



$$\text{Arcsin}' x = \frac{1}{\sqrt{1-x^2}}$$

⇒ Arcsin de classe $\mathcal{C}^\infty$

- $g: \mathbb{R} \rightarrow \mathbb{R}, x \mapsto \text{Arcsin} \circ \sin(x)$; g est périodique, impaire et $g(\frac{\pi}{2}) = \frac{\pi}{2}$

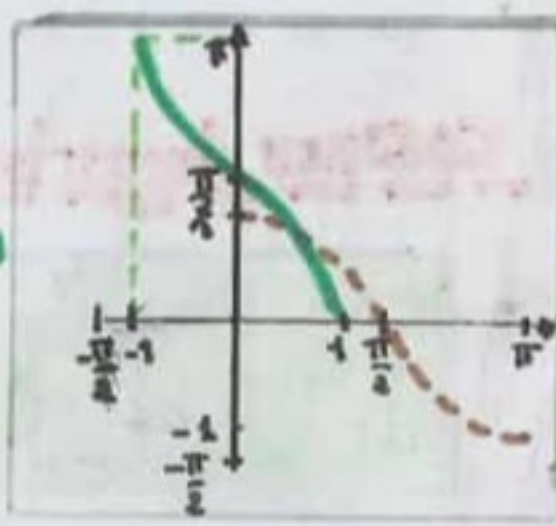


$$g(x) = \begin{cases} x & \text{si } x \in [0, \frac{\pi}{2}] \\ \pi - x & \text{si } x \in [\frac{\pi}{2}, \pi] \end{cases}$$

Arccos

cos: green, arccos: dark green

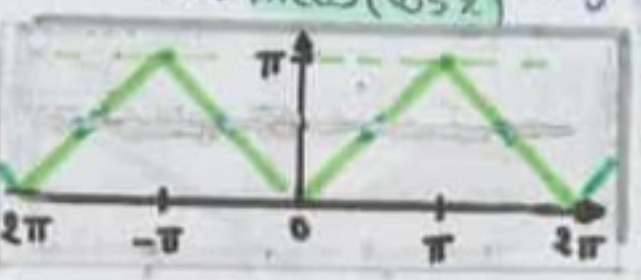
- $\forall x \in [-1, 1], \text{Arccos}(x) \in [0, \pi]$
- $\cos \circ \text{Arccos}(x) = x, \forall x \in [-1, 1]$
- Arccos n'est ni paire ni impaire
- $\text{Arccos}(x) + \text{Arccos}(-x) = \pi$
- Arccos(x) est dérivable sur $] -1, 1 [$



$$\text{Arccos}' x = \frac{-1}{\sqrt{1-x^2}}$$

⇒ Arccos de classe $\mathcal{C}^\infty$

- $g: \mathbb{R} \rightarrow \mathbb{R}, x \mapsto \text{Arccos}(\cos x)$; g est périodique, paire et $g(\pi) = \pi$



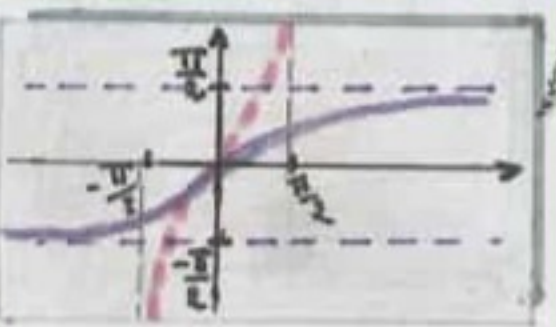
$$g(x) = \begin{cases} x & \text{si } x \in [0, \pi] \\ 2\pi - x & \text{si } x \in [\pi, 2\pi] \end{cases}$$

$$\forall x \in [-1, 1]; \text{Arccos}(x) + \text{Arcsin}(x) = \frac{\pi}{2}$$

Arctan

tan: pink, arctan: purple

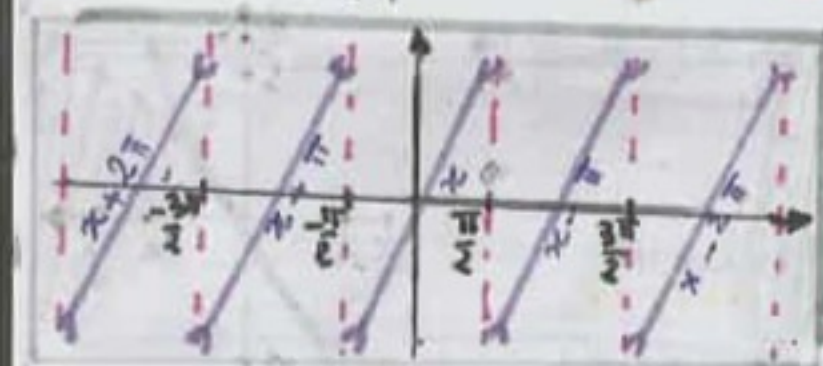
- $\forall x \in \mathbb{R}, \text{Arctan}(x) \in] -\frac{\pi}{2}, \frac{\pi}{2} [$
- Arctan est impaire
- $\tan \circ \text{Arctan}(x) = x, \forall x \in \mathbb{R}$
- Arctan(x) est dérivable sur $\mathbb{R}$



$$\text{Arctan}' x = \frac{1}{1+x^2}$$

$$\mathbb{R} : x \mapsto \text{Arctan}(\tan x) \text{ def } \mathbb{R} : \left[\frac{\pi}{2} + k\pi \right]$$

$$\text{R p\'eriodique, impaire } \forall x \in [0, \frac{\pi}{2}[, \text{R}(x) = x$$



$$\begin{aligned} \text{Si } x > 0; \\ \text{Arctan } x + \text{Arctan } \frac{1}{x} &= +\frac{\pi}{2} \\ \text{Si } x < 0; \\ \text{Arctan } x + \text{Arctan } \frac{1}{x} &= -\frac{\pi}{2} \end{aligned}$$

III. Fonctions hyperboliques:

Sinus hyperbolique:

$$\begin{aligned} \text{Sh} : \mathbb{R} &\rightarrow \mathbb{R} \\ x &\mapsto \frac{e^x - e^{-x}}{2} \end{aligned}$$

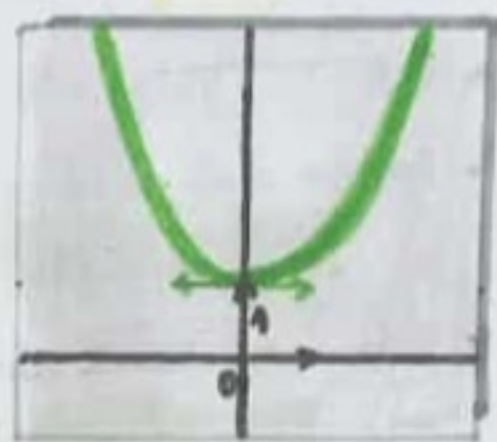
- Sh est une fct impaire et $\uparrow$



Cosinus hyperbolique:

$$\begin{aligned} \text{Ch} : \mathbb{R} &\rightarrow [1, +\infty[\\ x &\mapsto \frac{e^x + e^{-x}}{2} \end{aligned}$$

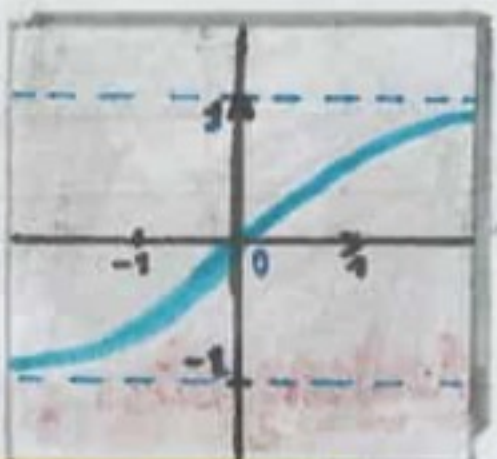
- Ch est une fct paire et $\uparrow$ pour $x > 0$



Tangente hyperbolique:

$$\begin{aligned} \text{Th} : \mathbb{R} &\rightarrow]-1, 1[\\ x &\mapsto \frac{\text{sh } x}{\text{ch } x} = \frac{e^{2x} - 1}{e^{2x} + 1} \end{aligned}$$

- Th croit de -1 à 1
- Th est impaire



$$\begin{aligned} &\bullet \text{ch } x + \text{sh } x = e^x \\ &\bullet \text{ch } x - \text{sh } x = e^{-x} \\ &\bullet \text{ch } x > \text{sh } x \quad \forall x \in \mathbb{R} \\ &\bullet \text{ch}^2 x - \text{sh}^2 x = 1 \\ &\bullet 1 - \text{th}^2 = \frac{1}{\text{ch}^2} = (\text{th } x)' \\ &\bullet (\text{ch } x)' = \text{sh } x \\ &\bullet (\text{sh } x)' = \text{ch } x \end{aligned}$$

IV. Fonctions hyperboliques R\'eciproques

Argsh:

$$\text{Argsh} : \mathbb{R} \rightarrow \mathbb{R}, x \mapsto \ln(x + \sqrt{x^2 + 1})$$

- fct impaire
- de classe e^∞
- strict $\uparrow$
- $\forall x \in \mathbb{R}, \text{Argsh}(\text{sh } x) = x$
- $\forall y \in \mathbb{R}, \text{sh}(\text{Argsh } y) = y$
- Argsh est d sur $\mathbb{R}$ et $\forall y \in \mathbb{R}$

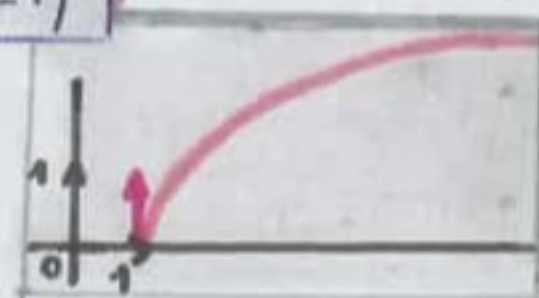


$$\text{Argsh}' y = \frac{1}{\sqrt{1+y^2}}$$

Argch:

$$\begin{aligned} \text{Argch} : [1, +\infty[&\rightarrow [0, +\infty[\\ x &\mapsto \ln(x + \sqrt{x^2 - 1}) \end{aligned}$$

- de classe e^∞
- strict $\uparrow$
- $\forall y \in [1, +\infty[, \text{ch}(\text{argch } y) = y$
- $\forall x \in [0, +\infty[, \text{argch}(\text{ch } x) = x$
- Argch est d sur $]1, +\infty[$ et $\forall y \in]1, +\infty[$



$$\text{Argch}' y = \frac{1}{\sqrt{y^2 - 1}}$$

Argth:

$$\begin{aligned} \text{Argth} :]-1, 1[&\rightarrow \mathbb{R} \\ x &\mapsto \frac{1}{2} \ln \left(\frac{x+1}{1-x} \right) \end{aligned}$$

- fct impaire
- de classe e^∞ sur $] -1, 1[$
- croit $\uparrow$
- Argth est d sur $] -1, 1[$ et $\forall y \in] -1, 1[$



$$\text{Argth}' y = \frac{1}{1-y^2}$$