

$$i^2 = -1$$

Nombres Complexes

$$(1+i)^2 = 2i$$

$$(1-i)^2 = -2i$$

* **Famille algébrique**

□ $z = a + ib$ avec $a = \operatorname{Re}(z)$ et $b = \operatorname{Im}(z)$

Conjugué d'un nombre complexe.

□ $z = 1 + 3i \rightarrow \bar{z} = 1 - 3i$

□ $z = -5i \rightarrow \bar{z} = 5i$

□ $z_1 = i - z \rightarrow \bar{z}_1 = -i - \bar{z}$

□ $z_2 = 1 - i z^2 \rightarrow \bar{z}_2 = 1 + i \bar{z}^2$

□ $z + \bar{z} = 2 \operatorname{Re}(z) = 2x$; $z - \bar{z} = 2i \operatorname{Im}(z) = 2iy$

$$z \bar{z} = \operatorname{Re}(z)^2 + \operatorname{Im}(z)^2 = x^2 + y^2 = |z|^2$$

□ $\bar{\bar{z}} = z$; $\bar{z}^2 = (\bar{z})^2$

Répère Complexe.

□ $M_1 = S_{(0, \vec{u})}(M)$

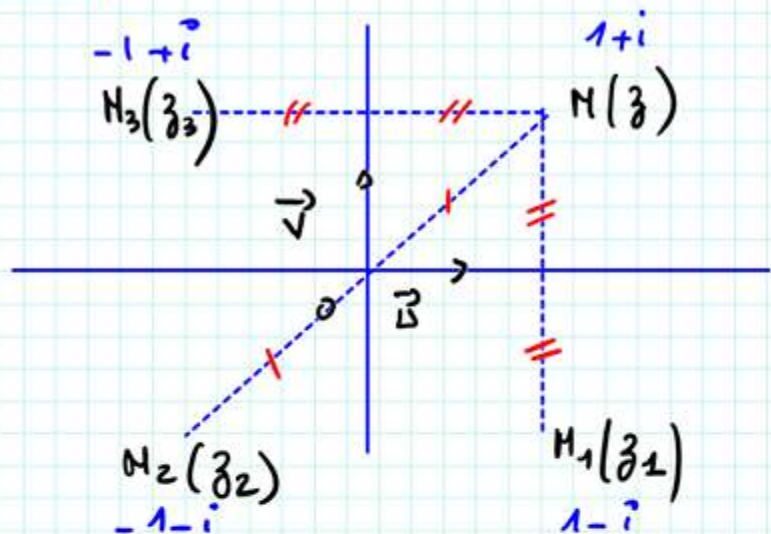
$\Rightarrow \boxed{z_1 = \bar{z}}$

□ $M_2 = S_O(M)$

$\Rightarrow \boxed{z_2 = -z}$

□ $M_3 = S_{(0, \vec{v})}(M)$

$\Rightarrow \boxed{z_3 = -\bar{z}}$



□ $z_A = x + iy \rightarrow A(x, y)$

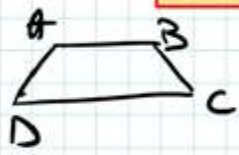
□ $z_M = x + iy \rightarrow M(x, y)$

□ $\boxed{z_{\vec{AB}} = z_B - z_A} ; \boxed{z_M = z_{\vec{OM}}}$

□ $ABCD$ est un plg $\Leftrightarrow z_{\vec{AB}} = z_{\vec{DC}}$
(A, B et C non alignés)

ou
 $A * C = B * D$

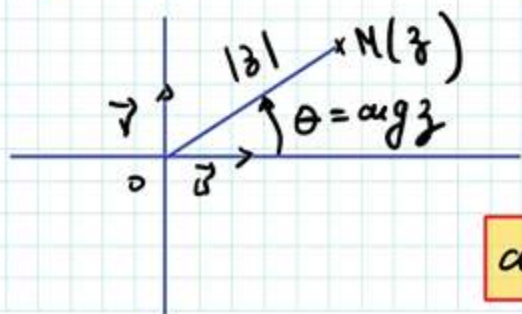
$\Leftrightarrow \frac{z_A + z_C}{2} = \frac{z_B + z_D}{2}$

- $ABCD$ est un losange $\Leftrightarrow \text{plg} + \boxed{AB = AD}$
- $ABCD$ est un rectangle $\Leftrightarrow \text{plg} + \boxed{\frac{z_{\vec{AB}}}{z_{\vec{AD}}} \in i\mathbb{R}}$
- $ABCD$ est un Carré $\Leftrightarrow \text{plg} + \boxed{AB = AD}$
 $+ \boxed{\frac{z_{\vec{AB}}}{z_{\vec{AD}}} \in i\mathbb{R}}$
- $ABCD$ est un trapèze 
 $\Rightarrow \frac{z_{\vec{AB}}}{z_{\vec{DC}}} \in \mathbb{R}$

Forme trigo - Forme expo

□ $z = |z| (\underbrace{\cos \theta + i \sin \theta}_{\text{F. Trigo}}) = |z| \underbrace{e^{i\theta}}_{\text{F. expo}}$ $z = x + iy$

$|z| = \sqrt{x^2 + y^2}$: module de z ; $\theta = \arg z [2\pi]$



$|z| = OM$

$(\vec{u}, \vec{ON}) \equiv \arg z [2\pi]$

$\arg(z_{\vec{AB}}) = (\vec{u}, \vec{AB}) [2\pi]$

$$\arg\left(\frac{z_{AB}}{z_{CB}}\right) = (\overrightarrow{CD}, \overrightarrow{AB}) [2\pi].$$

module

$$\square |z \times z'| = |z| \times |z'|$$

$$\square \left| \frac{z'}{z} \right| = \frac{|z'|}{|z|}$$

$$\square |z^n| = |z|^n$$

$$\square z \bar{z} = |z|^2 \neq z^2$$

argument.

$$\square \arg(z \times z') \equiv \arg z + \arg z' [2\pi]$$

$$\square \arg\left(\frac{z'}{z}\right) \equiv \arg z' - \arg z [2\pi]$$

$$\square \arg(z^n) \equiv n \arg z [2\pi]$$

$$\square \arg \bar{z} \equiv -\arg z [2\pi]$$

$$\arg(i\alpha) \equiv \frac{\pi}{2} [2\pi] ; \alpha \in \mathbb{R}_+^*$$

$$\arg(-i\alpha) \equiv -\frac{\pi}{2} [2\pi] ; \alpha \in \mathbb{R}_+^*$$

$$\arg(\alpha) \equiv 0 [2\pi] ; \alpha \in \mathbb{R}_+^*$$

$$\arg(\alpha) \equiv \pi [2\pi] ; \alpha \in \mathbb{R}_-^*$$

$$\arg(2017) \equiv 0 [2\pi] ; \arg(-2016) \equiv \pi [2\pi]$$

$$\arg(5i) \equiv \frac{\pi}{2} [2\pi] ; \arg(-2i) \equiv -\frac{\pi}{2} [2\pi]$$

$$\blacksquare \arg(1 - i\sqrt{3}) \equiv \theta [2\pi]$$

$$|1 - i\sqrt{3}| = \sqrt{1^2 + \sqrt{3}^2} = \sqrt{4} = 2$$

$$\begin{cases} \cos \theta = \frac{\operatorname{Re}(1 - i\sqrt{3})}{|1 - i\sqrt{3}|} = \frac{1}{2} \\ \sin \theta = \frac{\operatorname{Im}(1 - i\sqrt{3})}{|1 - i\sqrt{3}|} = \frac{-\sqrt{3}}{2} \end{cases} \quad \theta = -\frac{\pi}{3}.$$

① donc $\arg(1 - i\sqrt{3}) \equiv -\frac{\pi}{3} [2\pi].$

$$\underbrace{1 - i\sqrt{3}}_{\text{F. Alg}} = \underbrace{2 \left(\cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right) \right)}_{\text{F. Trig}} = \underbrace{2 e^{-i\frac{\pi}{3}}}_{\text{F. expo}}$$

$$\blacksquare e^{i\theta} = \cos \theta + i \sin \theta \quad \blacksquare |e^{i\theta}| = 1, \arg e^{i\theta} \equiv \theta [2\pi]$$

$$\begin{aligned} \blacksquare \arg(-e^{i\theta}) &\equiv \arg(-1) + \arg(e^{i\theta}) [2\pi] \\ &\equiv \pi + \theta [2\pi]. \end{aligned}$$

$$\blacksquare \arg(2e^{i\theta/2}) \equiv \frac{\theta}{2} [2\pi].$$

$$\blacksquare \arg(2i e^{i\theta}) \equiv \frac{\pi}{2} + \theta [2\pi]$$

$$\blacksquare \arg(-2i e^{i\theta}) \equiv -\frac{\pi}{2} + \theta [2\pi]$$

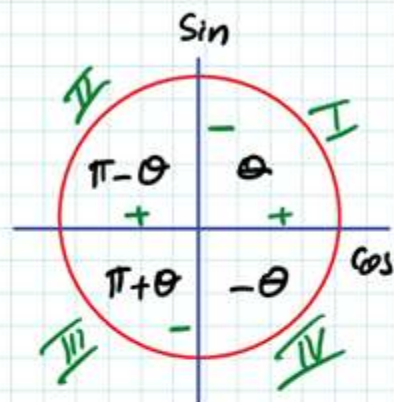
$$\square \arg(2i + e^{i\theta}) = ? [2\pi]$$

$$\text{I} \square \cos\theta + i\sin\theta = e^{i\theta}$$

$$\text{IV} \square \cos\theta - i\sin\theta = e^{-i\theta}$$

$$\text{II} \square -\cos\theta + i\sin\theta = e^{i(\pi-\theta)}$$

$$\text{III} \square -\cos\theta - i\sin\theta = e^{i(\pi+\theta)}$$



$$\square \arg(-\cos\theta + i\sin\theta) = \pi - \theta [2\pi]$$

$$\square \sin\theta + i\cos\theta = i(\cos\theta - i\sin\theta) = e^{i\frac{\pi}{2}} \cdot e^{-i\theta} = e^{i(\frac{\pi}{2}-\theta)}$$

$$\begin{aligned} e^{i\frac{\pi}{2}} &= i \\ e^{-i\frac{\pi}{2}} &= -i \\ e^{i0} &= 1 \\ e^{i\pi} &= -1 \end{aligned}$$

$$\begin{aligned} e^{i\theta} \cdot e^{i\theta'} &= e^{i(\theta+\theta')} \\ \frac{e^{i\theta'}}{e^{i\theta}} &= e^{i(\theta'-\theta)} \\ (e^{i\theta})^n &= e^{in\theta} \end{aligned}$$

$$2e^{i\frac{\pi}{3}} =$$

SHIFT (-)

Formule d'Euler.

- $2\cos\theta = e^{i\theta} + e^{-i\theta}$
- $2i\sin\theta = e^{i\theta} - e^{-i\theta}$
- $-2i\sin\theta = e^{-i\theta} - e^{i\theta}$
- $1 + e^{i\theta} = e^{i\frac{\theta}{2}} (e^{-i\frac{\theta}{2}} + e^{i\frac{\theta}{2}}) = 2\cos\frac{\theta}{2} e^{i\frac{\theta}{2}}$
- $1 - e^{i\theta} = e^{i\frac{\theta}{2}} (e^{-i\frac{\theta}{2}} - e^{i\frac{\theta}{2}}) = -2i\sin\frac{\theta}{2} e^{i\frac{\theta}{2}}$

Equation du 2^e degré

$$E: az^2 + bz + c = 0 \quad (a \neq 0)$$

- Si a, b et $c \in \mathbb{R}$ et $\Delta < 0$ alors E admet 2 solutions conjuguées

- Si E admet 2 solutions distincts z' et z'' on a:

$$z' + z'' = -\frac{b}{a}$$

$$\rightarrow \text{Si } b=0$$

E admet 2 solutions opposées.

$$z' \times z'' = \frac{c}{a}$$

$$\rightarrow \text{Si } a=c$$

E admet 2 solutions inverses.

$$\Delta = b^2 - 4ac = \delta^2 \quad \text{ou} \quad \Delta' = b'^2 - ac = \delta'^2 \quad b' = \frac{b}{2}$$

$$z' = \frac{-b - \delta}{2a}$$

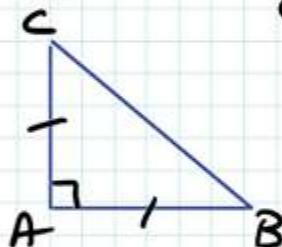
$$z' = \frac{-b' - \delta'}{a}$$

$$z'' = \frac{-b + \delta}{2a}$$

$$z'' = \frac{-b' + \delta'}{a}$$

Remarque importante dans la pratique.

□ Hq ABC est un triangle rectangle isocèle en A



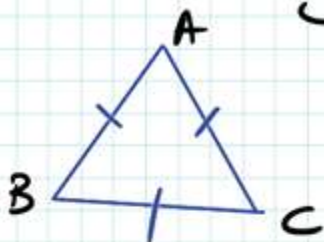
$$\frac{z_{\vec{AB}}}{z_{\vec{AC}}} = \frac{z_B - z_A}{z_C - z_A} = \pm i$$

En effet. $\frac{z_{\vec{AB}}}{z_{\vec{AC}}} \in i\mathbb{R}$ donc $\vec{AB} \perp \vec{AC}$ (1)

$$\left| \frac{z_{\vec{AB}}}{z_{\vec{AC}}} \right| = |\pm i| \Leftrightarrow \frac{AB}{AC} = 1 \Leftrightarrow AB = AC \quad (2)$$

(1) et (2) $\Rightarrow ABC$ est rectangle isocèle en A

□ Mg ABC est un triangle équilatéral



$$\frac{z_{\overrightarrow{AB}}}{z_{\overrightarrow{AC}}} = \frac{z_B - z_A}{z_C - z_A} = e^{\pm i \frac{\pi}{3}}$$

En effet : $\left| \frac{z_{\overrightarrow{AB}}}{z_{\overrightarrow{AC}}} \right| = 1 \Rightarrow \frac{AB}{AC} = 1$

$\Rightarrow AB = AC \quad (1)$

$$\begin{aligned} \arg\left(\frac{z_{\overrightarrow{AB}}}{z_{\overrightarrow{AC}}}\right) &\equiv \arg(z_{\overrightarrow{AB}}) - \arg(z_{\overrightarrow{AC}}) [2\pi] \\ &\equiv (\vec{u}, \overrightarrow{AB}) - (\vec{u}, \overrightarrow{AC}) [2\pi] \\ &\equiv (\overrightarrow{AC}, \vec{u}) + (\vec{u}, \overrightarrow{AB}) [2\pi] \\ &\equiv (\overrightarrow{AC}, \overrightarrow{AB}) [2\pi] \\ &\equiv \pm \frac{\pi}{3} [2\pi] \quad (2) \end{aligned}$$

(1) et (2) $\Rightarrow$ ABC est équilatéral

$$E: az^3 + bz^2 + cz + d = 0 \quad (a \neq 0)$$

Si z_0 solution de E , alors

$$az^3 + bz^2 + cz + d = (z - z_0)(az^2 + \alpha z + \beta)$$

α et β à déterminer.

$$E: (z - z_0)(az^2 + \alpha z + \beta) = 0.$$

$$\text{Soit } z - z_0 = 0 \text{ ou } az^2 + \alpha z + \beta = 0$$

$$z^n = re^{i\theta}.$$

$$\Leftrightarrow z_k = \sqrt[n]{r} e^{i\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right)}$$

avec $0 \leq k \leq n-1$